Qualifying Examination Syllabus

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1 Major Topic: Symplectic Geometry (Geometry/Topology)

1. Symplectic Linear Algebra
   (a) Symplectic vector spaces
   (b) Symplectic linear group
   (c) Lagrangian subspaces
   (d) Maslov index
   (e) Symplectic vector bundles
2. Symplectic Manifolds
   (a) Basic concepts
   (b) Moser isotopy and Darboux
   (c) Isotopy extension theorems
   (d) Tubular neighbourhood theorem
   (e) Submanifolds
   (f) Lagrangian submanifolds
   (g) Hamiltonian vector fields
3. Contact Manifolds
   (a) Contact structures
   (b) Reeb vector fields
   (c) Reeb vector fields and Hamiltonian vector fields
   (d) Symplectization
4. Almost-complex structures
   (a) Almost complex structures
   (b) Compatible triples
   (c) Integrability
   (d) Kahler manifolds
5. Symplectic group actions
   (a) Circle actions
   (b) moment maps
   (c) symplectic quotients by circle actions
6. J-holomorphic curves
   (a) Definitions
   (b) Transversality and somewhere injective curves
   (c) Fredholm index
   (d) Basic idea of Gromov compactness
   (e) Outline of proof of Gromov nonsqueezing

Relevant Texts 📘

Da Silva, Ana Cannas *Lectures on symplectic geometry.* Vol. 3575. Berlin: Springer, 2001.

McDuff, Dusa, and Dietmar Salamon. *Introduction to symplectic topology.* Vol. 27. Oxford University Press, 2017.

McDuff, Dusa, and Dietmar Salamon. *J-holomorphic curves and symplectic topology.* Vol. 52. American Mathematical Soc., 2012.

2 Major Topic: Differential Topology (Geometry/Topology)

1. Smooth Manifolds
   (a) Basic definitions
   (b) Maps between Manifolds
2. (Co)tangent spaces, (Co)tangent bundles and Vector Fields
   (a) Tangent bundles
   (b) Cotangent bundles
   (c) Vector fields and differential forms
   (d) Lie brackets
   (e) Integral curves and flows
   (f) distributions
   (g) Foliations
3. Submanifolds and transversality
   (a) Immersions, submersions and embeddings
4. Vector bundles, principal bundles, and fibre bundles
   (a) Vector bundle constructions
   (b) Sections of vector bundles
   (c) Bundle homomorphisms
   (d) fiber bundles
   (e) principal bundles
5. Calculus on Manifolds
   (a) Tensor algebra, symmetric and alternating tensors
   (b) Tensor fields
   (c) algebra of differential forms, p, q-tensors
   (d) Lie derivative, derivations, contractions
   (e) Cartan's magic formula
   (f) De Rham cohomology, DR cohomology of vector bundles
6. Integration + Stokes' Theorem
   (a) Integrating differential forms
   (b) Orientation
   (c) Stokes theorem
   (d) Manifolds with boundary and generalized Stokes' theorem
7. Connections
   (a) Connections,
   (b) Torsion and curvature, Bianchi identities
   (c) Parallel transport
   (d) Geodesics
8. Riemannian Geometry
   (a) Metrics
   (b) Levi-Civita connection
   (c) Riemann curvature
9. Morse Theory
   (a) Morse functions
   (b) Morse homology
   (c) Isomorphism between Morse homology and cellular homology
   (d) Morse inequalities

Relevant Texts

Lee, John M. *Introduction to Smooth Manifolds.* Springer, 2000.

Nicolaescu, Liviu I. *Lectures on the geometry of manifolds.* World Scientific Publishing, 2021.

Milnor, John. *Morse Theory.* Princeton University Press, 1963.

Hutchings, Michael. *Lecture notes on Morse homology (with an eye towards Floer theory and pseudoholomorphic curves.* UC Berkeley, 2002.

3 Minor Topic: Finite Dimensional and Semisimple Lie Algebras (Algebra)

1. Basic Notions
   (a) Definition of a Lie algebra
   (b) Simple and semisimple Lie algebras
2. Lie Algebra formulations and properties
   (a) Associative algebras
   (b) Classic examples:
   (c) Derivations
   (d) Representations
   (e) Solvable and Nilpotent Lie algebras
   (f) Lie's theorem, Engel's theorem
3. Invariant forms and Cartan-Killing Criterions
   (a) Invariant forms
   (b) Cartan's criterion for solubility
   (c) Cartan-Killing's criterion for semisimplicity
   (d) Jordan decomposition
4. Root systems
   (a) Coxeter graphs and Dynkin diagrams
   (b) Constructing root systems from Cartan matrices and Dynkin diagrams
5. Representation Theory of Semisimple Complex Lie algebras
   (a) Weyl's theorem
   (b) Casimir elements
   (c) The Universal enveloping algebra
   (d) Statement of Poincaré-Birkhoff-Witt theorem

Relevant Texts

J.E. Humphries, *Introduction to Lie algebras and representation theory.* Springer, 1972.

K. Erdmann and M.J. Wildon, *Introduction to Lie algebras.* Springer, 2006.

R.S. Pierce, *Associative algebras.* Springer, 1982.